Augmented Designs

### Properties

- $t = \text{number of unreplicated treatments}$
- $r = \text{number of blocks}$
- $n = \frac{t}{r} = \text{number of unreplicated treatments per rep}$
- $c = \text{number of replicated checks}$
- $p = c + n = \text{number of plots per rep}$
- $N = rc + t = r(c + n) = \text{total number of plots}$
Critical F Values for Alpha 0.05
Numerator df = 1

$\text{error df} = (r - 1)(c - 1)$

Assuming a minimum of 10 df is need for error:

$r \geq \frac{10}{c - 1} + 1$

for $c = 4$:

$r \geq \frac{10}{4 - 1} + 1 = 4.3$

So 5 reps are required
Augmented Designs

Soybean Evaluation Example

40 Selections – limited seed quantity

Potential designs:

<table>
<thead>
<tr>
<th>r</th>
<th>n</th>
<th>c</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>10</td>
<td>10/(4 - 1) + 1 = 5</td>
<td>4(5) + 40 = 60</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>10/(5 - 1) + 1 = 4</td>
<td>5(4) + 40 = 60</td>
</tr>
</tbody>
</table>

Augmented Designs

Soybean Evaluation Example

<table>
<thead>
<tr>
<th>Rep</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>C4</td>
</tr>
<tr>
<td>3</td>
<td>C2</td>
</tr>
<tr>
<td>4</td>
<td>C2</td>
</tr>
<tr>
<td>5</td>
<td>C1</td>
</tr>
</tbody>
</table>
Augmented Designs
Soybean Evaluation Example

<table>
<thead>
<tr>
<th>Rep</th>
<th>Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>C4</td>
</tr>
<tr>
<td>3</td>
<td>40 C2</td>
</tr>
<tr>
<td>4</td>
<td>33 C2</td>
</tr>
<tr>
<td>5</td>
<td>37 C1</td>
</tr>
</tbody>
</table>

Augmented Designs
Analysis

Steps:
1. Run ANOVA on checks
2. Calculate block adjustment factor $a$:
   \[ a_j = \bar{x}_j - \bar{x} \cdot \sum_j a_j = 0 \]
3. Adjust treatment means:
   \[ \hat{y}_{ij} = y_{ij} - a_j \]
Augmented Designs
Analysis

Standard Errors

Differences in check means:
\[ S_\alpha = \sqrt{\frac{2MSE}{r}} \]

Differences in adjusted treatment means from the same block:
\[ S_\alpha = \sqrt{2MSE} \]

Differences in adjusted treatment means from different blocks:
\[ S_\alpha = \sqrt{\frac{2(c+1)MSE}{c}} \]

Difference between an adjusted treatment mean and check mean:
\[ S_\alpha = \sqrt{\frac{(r+1)(c+1)MSE}{rc}} \]
Augmented Designs
Wheat Example
Peterson, 1994, p. 170

**Design:**
\[ t = 30 \text{ wheat selections} \]
\[ c = 3 \text{ checks} \]
\[ r = 6 \text{ reps} \]
\[ n = t/r = 5 \text{ selections/rep} \]
\[ p = c + n = 8 \text{ plots per rep} \]
\[ N = rc + t = 48 \text{ total plots} \]

---

**ANOVA**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>5</td>
<td>6968486</td>
<td>1393697</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check</td>
<td>2</td>
<td>20050</td>
<td>10025.39</td>
<td>0.11</td>
<td>0.8969</td>
</tr>
<tr>
<td>Error</td>
<td>10</td>
<td>911026</td>
<td>91102.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>17</td>
<td>7899563</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Augmented Designs
Wheat Example

Block Adjustment Factors

<table>
<thead>
<tr>
<th>Check</th>
<th>Block 1</th>
<th>Block 2</th>
<th>Block 3</th>
<th>Block 4</th>
<th>Block 5</th>
<th>Block 6</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cimmaron</td>
<td>2592</td>
<td>3023</td>
<td>2918</td>
<td>2940</td>
<td>1398</td>
<td>3483</td>
<td>2725.7</td>
</tr>
<tr>
<td>Stork</td>
<td>2972</td>
<td>3122</td>
<td>2260</td>
<td>3348</td>
<td>1315</td>
<td>3538</td>
<td>2759.2</td>
</tr>
<tr>
<td>Waha</td>
<td>2608</td>
<td>2477</td>
<td>3107</td>
<td>2850</td>
<td>1625</td>
<td>3400</td>
<td>2677.8</td>
</tr>
<tr>
<td>Mean</td>
<td>2724.0</td>
<td>2874.0</td>
<td>2761.7</td>
<td>3046.0</td>
<td>1446.0</td>
<td>3473.7</td>
<td>2720.9</td>
</tr>
<tr>
<td>a</td>
<td>3.1</td>
<td>153.1</td>
<td>40.8</td>
<td>325.1</td>
<td>-1274.9</td>
<td>752.8</td>
<td></td>
</tr>
</tbody>
</table>

Augmented Designs
Wheat Example

Standard Errors

Differences in check means:

$$S_d = \sqrt{\frac{2(91,103)}{6}} = 174$$

Differences in adjusted treatment means from the same block:

$$S_{\bar{d}} = \sqrt{2(91,103)} = 427$$
Augmented Designs
Wheat Example

Standard Errors

Differences in adjusted treatment means from different blocks:

\[ S_d = \sqrt{\frac{2(3+1)91,103}{3}} = 493 \]

Difference between an adjusted treatment mean and check mean:

\[ S_d = \sqrt{\frac{(6+1)(3+1)91,103}{6(3)}} = 376 \]

Augmented Designs
SAS Analysis

```sas
proc mixed;
  class rep trt;
  model yld = rep trt;
  lsmeans trt / pdiff;
run;
```
Augmented Designs

SAS Analysis

**proc mixed data=a;**
```sas
  class block select;
  model yield = block select;
  estimate 'Sel in same Block' select -1 0 0 1;
  estimate 'Sel in dif Block' select -1 1;
  estimate 'Check means' select 0 0 0 0 0 0 0 0 0 0 0 -1 1;
  estimate 'Check v. Sel means' select 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 1;
run;
```

**Type 3 Tests of Fixed Effects**

<table>
<thead>
<tr>
<th>Effect</th>
<th>Num DF</th>
<th>Den DF</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>block</td>
<td>5</td>
<td>10</td>
<td>15.30</td>
<td>0.0002</td>
</tr>
<tr>
<td>select</td>
<td>32</td>
<td>10</td>
<td>1.40</td>
<td>0.2930</td>
</tr>
</tbody>
</table>

**Estimates**

<table>
<thead>
<tr>
<th>Label</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>DF</th>
<th>t Value</th>
<th>Pr &gt;</th>
<th>t</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sel in same Block</td>
<td>-628.00</td>
<td>426.86</td>
<td>10</td>
<td>-1.47</td>
<td>0.1720</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sel in dif Block</td>
<td>307.67</td>
<td>492.89</td>
<td>10</td>
<td>0.62</td>
<td>0.5465</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check means</td>
<td>33.50</td>
<td>174.26</td>
<td>10</td>
<td>0.19</td>
<td>0.8514</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check v. Sel means</td>
<td>782.78</td>
<td>362.76</td>
<td>10</td>
<td>2.16</td>
<td>0.0563</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mean Comparisons

Yield (kg/ha)

Line